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The triality of electromagnetic-condensational waves in a gas-like ether

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Abstract In a gas-like ether, the duality between the oscillating electric and magnetic fields, which are transverse to the direction of propagation of electromagnetic waves, becomes a triality with the longitudinal oscillations of motion of the ether, if electric field, magnetic field and motion are coexistent and mutually perpendicular. It must be shown, therefore, that if electromagnetic waves comprise also longitudinal condensational oscillations of a gas-like ether, analogous to sound waves in a material gas, then all three aspects of such waves must propagate together along identical wave-fronts. To this end, the full characteristic hyperconoids are derived for the equations governing the motion and the electric and magnetic field-strengths in a gas-like ether, in three space variables and time. It is shown that they are, in fact, identical. The equations governing the motion and the electric and magnetic field-strengths in such an ether, and their common characteristic hyperconoid, are all invariant under Galilean transformation.

1 Introduction

It has been shown[1] that Planck's energy distribution for black-body radiation can be derived for an ethereal medium which behaves as an ideal gas with Maxwellian statistics. Electromagnetic waves may propagate in such an ether, and the oscillating electric and magnetic fields in such waves are observed to be transverse to the direction of wave propagation. On the other hand, electric field, magnetic field and motion are generally observed to be mutually perpendicular and coexistent, and this suggests that such waves must also comprise longitudinal oscillations in pressure and density. Thus, such waves would not merely have the duality of being electromagnetic, but rather the triality of being electromagnetic-condensational waves, and their condensational aspect would be analogous to that of sound waves in a material gas.

To justify such a concept of the ether, it is, therefore, necessary to establish such a triality of electromagnetic-condensational waves, by showing that all three aspects of the waves propagate together contemporaneously along precisely the same wave-fronts. In general, for three space-variables and time, the wave fronts are given by the characteristic hypersurfaces of the partial differential equations which govern the electric and magnetic field strengths and the motion of such an ether. All such hypersurfaces which pass through a given point in space-time have an envelope, the characteristic hyperconoid through the point.

It is the purpose here to derive the characteristic hyperconoid both for the equations of electricity and magnetism in a gas-like ether, and for the general equations governing the unsteady motion of a gas in three space-variables, and thus to show that they are, in fact, identical. Such a concept of the ether entails no transformation difficulties. For the equations of electricity and magnetism in a gas-like ether, the general equations of unsteady motion of a gas, and their common characteristic hyperconoid, are all invariant under Galilean transformation. Moreover, with such a concept of the ether, there is no dichotomy between the observed wave and particle properties of radiation, for these are essentially no different from the wave and particle properties of sound in a material gas.

2 Characteristic hypersurfaces

Consider a system of m simultaneous linear first-order partial differential equations in n independent variables $x_i (i=1,...,n)$, with m unknowns $u_\alpha (\alpha=1,...,m)$ whose coefficients $a_{i,\alpha\beta}$ and non-differential terms b_α are all functions of the values of x_i and u_α. With the summation convention, such a system may be written as

$$a_{i,\alpha\beta}\frac{\partial u_\beta}{\partial x_i}=b_\alpha \tag{2.1}$$

If the values of u_α are known at all points on a hypersurface $\zeta(x_i)=$ const. $=K$ then, in general, the partial differential equations serve to determine the derivatives $du_\alpha/d\zeta$ at all points of the hypersurface, and thus the values of u_α on the neighbouring hypersurface $\zeta(x_i)=K+dK$. For the equations Eq.(2.1) can be rewritten in the form

$$\left(a_{i,\alpha\beta}\frac{\partial \zeta}{\partial x_i}\right)\frac{du_\beta}{d\zeta}=b_\alpha \tag{2.2}$$

The hypersurface $\zeta(x_i)=K$ is said to be characteristic[2] if Eqs (2.2) are indeterminate, in which case $\zeta(x_i)$ must satisfy the determinantal equation

$$\left|a_{i,\alpha\beta}\frac{\partial \zeta}{\partial x_i}\right|=0 \tag{2.3}$$

Alternatively,[3] the hypersurface $\zeta(x_i)=K$ is said to be characteristic if there is a linear combination of Eqs (2.1) which involves only total derivatives with respect to ζ. In order that the linear combination of Eqs (2.1), namely

$$\lambda_\alpha\left(a_{i,\alpha\beta}\frac{\partial \zeta}{\partial x_i}\right)\frac{du_\beta}{d\zeta}=\lambda_\alpha b_\alpha$$

may involve only total derivatives with respect to ζ, it follows that

$$\lambda_\alpha a_{i,\alpha\beta}\frac{\partial \zeta}{\partial x_i}=f_\beta \tag{2.4}$$

for some function f_β of the values of x_i and u_α. Equations (2.4) are m equations to determine the $(m-1)$ independent ratios between the λ_α values and so must be indeterminate. Thus, again, the condition for $\zeta(x_i)=K$ to be a characteristic hypersurface is identical with Eq. (2.3), and so the two definitions of a characteristic hypersurface are equivalent. In general, the characteristic hypersurfaces which pass through any point $\{x_i\}$ have an envelope in the form of a hyperconoid. This may be determined in terms of differential displacements, dx_i, from the point, since any such displacement which lies in the hypersurface, $\zeta(x_i)=K$ satisfies

$$d\zeta = \frac{\partial \zeta}{\partial x_i} dx_i = 0 \tag{2.5}$$

3 The characteristic hyperconoid of the ethereal motion

For one-dimensional unsteady motion of a gas, whose speed is denoted by u, it is well-known that the two characteristic curves through any point, which correspond to the wave-fronts, are given by

$$\frac{dx}{dt} = u \pm c \quad \text{or} \quad (dx - udt)^2 - c^2(dt)^2 = 0 \tag{3.1}$$

where c denotes the local wave-speed.

For unsteady flow of a gas in two space-variables, x_1 and x_2, when the velocity components are denoted by u_1 and u_2 respectively, the characteristic hyperconoid for the wave-fronts is derived[2] as

$$(dx_1 - u_1dt)^2 + (dx_2 - u_2dt)^2 - c^2(dt)^2 = 0 \tag{3.2}$$

The results, Eqs (3.1) and (3.2), both express the same simple physical property, namely that waves of infinitesimal amplitude propagate in all possible directions at the local wave-speed c relative to the local fluid motion. The extension to three space-variables may be inferred very easily but, nevertheless, a full derivation is given here. For general unsteady motion of a gas in three space-variables $x_i, (i=1,2,3)$ when the fluid velocity components are denoted by u_i, the governing equations may be written, again using the summation convention,

$$\text{(Mass)} \quad \frac{Dv}{Dt} - v\frac{\partial u_i}{\partial x_i} = -Av^2 \tag{3.3}$$

$$\text{(Momentum)} \quad \frac{Du_i}{Dt} + v\frac{\partial p}{\partial x_i} = B_i v \tag{3.4a}$$

$$\text{(Energy)} \quad \frac{DS}{Dt} = \frac{v}{T}(H - Apv) \tag{3.5}$$

Here p denotes, pressure, v specific volume, S specific entropy, T absolute temperature and the total time-derivative, moving with the fluid, is given by

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} \tag{3.6}$$

A, B_i and H denote, respectively, any external sources of mass, momentum and energy, per unit volume per unit time.

Equations (3.3), (3.4a) and (3.5) are a simultaneous system of five equations of the type considered in Section 2 above, in the four independent variables x_i, t, with five unknowns, namely $\{u_i\}$ and two independent thermodynamic variables. In the (E, v, S) system of thermodynamics, with v and S as the two independent thermodynamic variables,[4] p may be expressed in terms of v and S, in Eqs (3.4a), by means of the thermodynamic identities

$$p \equiv -E_v, \quad c^2 \equiv v^2 E_{vv}, \quad \Gamma \equiv -v\frac{E_{vS}}{E_S}, \quad T \equiv E_S \tag{3.7}$$

where E is the intrinsic energy per unit mass, Γ is the Grüneisen index, and suffixes v and S denote partial differentiation. For then

$$\frac{\partial p}{\partial x_i} = -\frac{\partial E_v}{\partial x_i} = -E_{vv}\frac{\partial v}{\partial x_i} - E_{vS}\frac{\partial S}{\partial x_i}$$

and so

$$\frac{\partial p}{\partial x_i} = -\left(\frac{c^2}{v^2}\right)\frac{\partial v}{\partial x_i} + \left(\frac{\Gamma T}{v}\right)\frac{\partial S}{\partial x_i} \tag{3.8}$$

allowing the three equations of Eqs (3.4a) to be written

$$\frac{Du_i}{Dt} - \left(\frac{c^2}{v}\right)\frac{\partial v}{\partial x_i} + \Gamma T\frac{\partial S}{\partial x_i} = B_i v \tag{3.4b}$$

The condition of Eq. (2.3) for the hypersurface $\zeta(x_i) = K$ to be characteristic is then

$$\begin{vmatrix} -v\frac{\partial\zeta}{\partial x_1} & -v\frac{\partial\zeta}{\partial x_2} & -v\frac{\partial\zeta}{\partial x_3} & \frac{D\zeta}{Dt} & 0 \\ \frac{D\zeta}{Dt} & 0 & 0 & -\left(\frac{c^2}{v}\right)\frac{\partial\zeta}{\partial x_1} & \Gamma T\frac{\partial\zeta}{\partial x_1} \\ 0 & \frac{D\zeta}{Dt} & 0 & -\left(\frac{c^2}{v}\right)\frac{\partial\zeta}{\partial x_2} & \Gamma T\frac{\partial\zeta}{\partial x_2} \\ 0 & 0 & \frac{D\zeta}{Dt} & -\left(\frac{c^2}{v}\right)\frac{\partial\zeta}{\partial x_3} & \Gamma T\frac{\partial\zeta}{\partial x_3} \\ 0 & 0 & 0 & 0 & \frac{D\zeta}{Dt} \end{vmatrix} = 0 \tag{3.9}$$

and this reduces to

$$\left(\frac{D\zeta}{Dt}\right)^3\left[\left(\frac{D\zeta}{Dt}\right)^2 - c^2\delta_{ij}\frac{\partial\zeta}{\partial x_i}\frac{\partial\zeta}{\partial x_j}\right] = 0 \tag{3.10}$$

where δ_{ij} is the Kronecker delta function $(=0, i \neq j; =1, i = j)$.

The relation (2.5) satisfied by a displacement in the hypersurface $\zeta(x_i) = K$ may be written in the form

$$\frac{D\zeta}{Dt} + \left(\frac{\partial\zeta}{\partial x_i}\right)\left(\frac{dx_i}{dt} - u_i\right) = 0 \tag{3.11}$$

It is clear from Eq. (3.11) that the vanishing of the first repeated factor $D\zeta/Dt$ in Eq. (3.10) corresponds to the world-lines

$$\frac{dx_1}{du_1} = \frac{dx_2}{du_2} = \frac{dx_3}{du_3} = dt$$

which must, therefore, be characteristic in the sense of the mathematical definitions given above. The vanishing of the second factor in Eq.(3.10) which corresponds to the wave-fronts, may, with the aid of Eq.(3.11), be written in a form independent of $\partial\zeta/\partial t$, namely

$$\left[\frac{\partial\zeta}{\partial x_i}\left(dx_i - u_i dt\right)\right]^2 = c^2 \delta_{ij} \frac{\partial\zeta}{\partial x_i}\frac{\partial\zeta}{\partial x_j}\left(dt\right)^2 \tag{3.12}$$

The characteristic hyperconoid is thus the envelope of the displacements (dx_i, dt) given by Eq. (3.12) in terms of the three parameters $\partial\zeta/\partial x_i$. This envelope must, therefore, satisfy, in addition to Eq. (3.12), the three following relations obtained by differentiating Eq. (3.12) successively with respect to these three parameters, *viz*:

$$\frac{\partial\zeta}{\partial x_i}\left(dx_i - u_i dt\right)\left(dx_k - u_k dt\right) = c^2 \frac{\partial\zeta}{\partial x_k}(dt)^2 \tag{3.13}$$

By squaring and adding both sides of the three relations of Eq. (3.13) and making use of Eq. (3.12), the characteristic hyperconoid is finally obtained as

$$\delta_{ij}\left(dx_i - u_i dt\right)\left(dx_j - u_j dt\right) - c^2(dt)^2 = 0$$

or

$$\left(dx_1 - u_1 dt\right)^2 + \left(dx_2 - u_2 dt\right)^2 + \left(dx_3 - u_3 dt\right)^2 - c^2(dt)^2 = 0 \tag{3.14}$$

4 The characteristic hyperconoid of the equations of electricity and magnetism

In the assembly of Maxwell's equations, the time-derivatives which occur in Ampère's rule and in the laws of induction have invariably been interpreted as the partial derivative $\partial/\partial t$. This is not acceptable in the concept of a gas-like ethereal medium, where the ethereal velocity may vary from point to point and with time, and the Newtonian frame of reference may be chosen so that its origin moves at any constant speed, independent of the ethereal motion. To satisfy the requirements of a gas-like ether unambiguously, the time-derivative in Ampère's rule and the laws of induction can only be interpreted as the total time-derivative moving with the ethereal flow, namely D/Dt, as defined in Eq. (3.6) above. The equations for the electric and magnetic field-strengths in a gas-like ether become then,[5] again using the summation convention ($i = 1, 2, 3$),

$$\frac{\partial E_i}{\partial x_i} = 0 \tag{4.1}$$

$$\frac{\partial H_i}{\partial x_i} = 0 \tag{4.2}$$

$$\epsilon_0 \frac{DE_i}{Dt} = \varepsilon_{ijk} \frac{\partial H_k}{\partial x_j} \tag{4.3}$$

$$\mu_0 \frac{DH_i}{Dt} = -\varepsilon_{ijk} \frac{\partial E_k}{\partial x_j} \tag{4.4}$$

Here $\{E_i\}$ is the electric field-strength, $\{H_i\}$ the magnetic field-strength, μ_0 is the magnetic permeability of the ether, ϵ_0 the permittivity and ε_{ijk} the alternating tensor. $(\mu_0\epsilon_0)^{-\frac{1}{2}}$ has the dimensions of a speed and is found observationally to be equal to c, the local wave-speed in the ether.

The eight equations of Eqs (4.1) – (4.4) are not independent, and are, essentially, a simultaneous system of six equations of the type discussed in Section (2) above, in the four independent variables (x_i) and t, with six unknowns, namely $\{E_i\}$ and $\{H_i\}$. (The components u_i of the ethereal velocity are determined by the governing equations, Eqs. (3.3), (3.4) and (3.5), of the ethereal motion.) The condition (2.3) for the hypersurface $\zeta(x_i) = K$ to be characteristic necessitates now the vanishing of all the sixth-order determinants of the matrix

$$\left\|\begin{array}{cccccc}
\frac{\partial\zeta}{\partial x_1} & \frac{\partial\zeta}{\partial x_2} & \frac{\partial\zeta}{\partial x_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\partial\zeta}{\partial x_1} & \frac{\partial\zeta}{\partial x_2} & \frac{\partial\zeta}{\partial x_3} \\
\epsilon_0\frac{D\zeta}{Dt} & 0 & 0 & 0 & -\frac{\partial\zeta}{\partial x_3} & \frac{\partial\zeta}{\partial x_2} \\
0 & \epsilon_0\frac{D\zeta}{Dt} & 0 & \frac{\partial\zeta}{\partial x_3} & 0 & -\frac{\partial\zeta}{\partial x_1} \\
0 & 0 & \epsilon_0\frac{D\zeta}{Dt} & -\frac{\partial\zeta}{\partial x_2} & \frac{\partial\zeta}{\partial x_1} & 0 \\
0 & \frac{\partial\zeta}{\partial x_3} & -\frac{\partial\zeta}{\partial x_2} & \mu_0\frac{D\zeta}{Dt} & 0 & 0 \\
-\frac{\partial\zeta}{\partial x_3} & 0 & \frac{\partial\zeta}{\partial x_1} & 0 & \mu_0\frac{D\zeta}{Dt} & 0 \\
\frac{\partial\zeta}{\partial x_2} & -\frac{\partial\zeta}{\partial x_1} & 0 & 0 & 0 & \mu_0\frac{D\zeta}{Dt}
\end{array}\right\|$$

Since the eight equations in six unknowns are self-consistent, all but one of the sixth order determinants of the above matrix vanish identically, and the condition for the hypersurface $\zeta(x_i) = K$ to be characteristic is found to reduce to

$$\left(\frac{D\zeta}{Dt}\right)^2\left[\left(\frac{D\zeta}{Dt}\right)^2-\left(\frac{1}{\mu_0\epsilon_0}\right)\delta_{ij}\frac{\partial\zeta}{\partial x_i}\frac{\partial\zeta}{\partial x_j}\right]^2=0 \tag{4.5}$$

As in Section 3 above, the vanishing of the first repeated factor in Eq. (4.5) corresponds to the world-lines. The second factor is now also repeated, as may be expected from the duality of the Eqs (4.1) – (4.4) and, exactly as in Section 3 above, it leads to precisely the same characteristic hyperconoid (3.14) when c^2 is written for $(\epsilon_0\mu_0)^{-1}$. Alternatively, Eqs (3.3) – (3.5) and (4.1) – (4.4) may be combined to form a system of essentially eleven equations of the type discussed in Section 2 above, in the eleven unknowns $\{u_i\}, v, S, \{E_i\}, \{H_i\}$, and the vanishing of the pertinent eleventh-order determinant would then give, as the condition for the hypersurface $\zeta(x_i)=K$ to be characteristic,

$$\left(\frac{D\zeta}{Dt}\right)^5\left[\left(\frac{D\zeta}{Dt}\right)^2-c^2\delta_{ij}\frac{\partial\zeta}{\partial x_i}\frac{\partial\zeta}{\partial x_j}\right]^3=0 \tag{4.6}$$

The triality of electromagnetic-condensational waves is now displayed by the triple occurrence of the second factor in Eq. (4.6).

5 Galilean transformation

Let (x_i,t) denote one non-rotating Newtonian frame of reference and (x_i',t') a second such frame of reference whose origin O' is at $\{X_i\}$ in the first frame ($\dot{X}_i$ is constant), and whose axes $O'x_i'$ have fixed direction cosines L_{ij} in the first frame. Then the Galilean transformation between the two frames of reference is, again using the summation convention ($i=1,2,3$),

$$x_i=X_i+L_{ji}x_j'; \quad x_i'=L_{ij}\left(x_j-X_j\right); \quad t=t' \tag{5.1}$$

The ethereal velocity components in the two frames of reference are connected by the relations

$$u_i=\dot{X}_i+L_{ji}u_j'; \quad u_i'=L_{ij}\left(u_j-\dot{X}_j\right) \tag{5.2}$$

and the wave-speed remains unchanged, so that

$$c'(x_i',t')=c(x_i,t) \tag{5.3}$$

Then, from Eq. (5.1)

$$dx_i=\dot{X}_i dt+L_{ji}dx_j'; \quad dt=dt' \tag{5.4}$$

whence, using Eq. (5.2)

$$\left(dx_i-u_i dt\right)=L_{ji}\left(dx_j'-u_j'dt'\right) \tag{5.5}$$

By squaring and adding both sides of the three relations of Eq.(5.5) it is found that

$$\delta_{ij}\left(dx_i - u_i dt\right)\left(dx_j - u_j dt\right) = \delta_{ij}\left(dx'_i - u'_i dt'\right)\left(dx'_j - u'_j dt'\right) \tag{5.6}$$

Equations (5.6) and (5.3) then show that the characteristic hyperconoid (3.14) is invariant under the transformation (5.1).

It is already well-known that the governing equations of motion, Eqs (3.3) – (3.5) are invariant under the transformation (5.1); likewise it is easily verified that Eqs (4.1) – (4.4) for the electric and magnetic field-strengths are also invariant under the transformation (5.1). For the differential relations, Eq. (5.4), lead to

$$\frac{\partial}{\partial x'_j} \equiv L_{ji}\frac{\partial}{\partial x_i}; \quad \frac{\partial}{\partial t'} \equiv \dot{X}_i \frac{\partial}{\partial x_i} + \frac{\partial}{\partial t} \tag{5.7}$$

whence, using Eq. (5.2) it is found that

$$\frac{D}{Dt'} \equiv \frac{D}{Dt} \tag{5.8}$$

6 Local approximations

If a frame of reference is chosen so that it moves at the same speed as the ether there, then, in the limit as the origin is approached, $\{u_i\} \to 0$, Eqs (4.1) – (4.4) governing the electric and magnetic field-strengths tend to Maxwell's equations, and the common characteristic hyperconoid tends to the limiting form

$$\left(dx_1\right)^2 + \left(dx_2\right)^2 + \left(dx_3\right)^2 - c^2\left(dt\right)^2 = 0 \tag{6.1}$$

Such limiting forms at the origin may serve as good approximations in the vicinity of the origin, to an extent which depends on the smallness of the gradients in $\{u_i\}$ near the origin.

There is no reason, however, why such local approximations should be invariant under transformation from one frame of reference to another. But clearly this may be achieved in practice by dropping the local approximation before transforming, and then restoring the local approximation near the origin of a new frame of reference, also chosen so that its origin moves with the local ether.

References

1. Thornhill, C.K., 'The kinetic theory of electromagnetic radiation', *Speculations Sci. Technol., 8*, 263 (1985).
2. Thornhill, C.K., *Aero. Res. Council Repts and Memoranda*, No. 2615 (1952).
3. Courant, R., and Friedrichs, K.O., *Supersonic Flow and Shock Waves*, Interscience, New York and London (1948).
4. Swan, G.W., and Thornhill, C.K., *Mech. Phys. Solids, 22*, 349 (1974).
5. Bleaney, B.I., and Bleaney, B., *Electricity and Magnetism*, Oxford University Press (1976).